

MILP for the inventory and routing problem for replenishment the car assembly line.

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Abstract: The inbound logistics for feeding the workstation inside the factory represents a critical issue in the car manufacturing industry. Nowadays, this issue is even more critical than in the past since more types of cars are being produced in the assembly lines. Consequently, as workstations have to install many types of components, they also need to have an inventory of different types of components in a usually compact space. The replenishment of inventory is a critical issue since a lack of it could cause line stoppage or reprocessing. On the other hand, an excess of inventory could increase the holding cost or even block the replenishment paths. The decision of the replenishment routes cannot be made without taking into consideration the inventory needed in each station during the production time, which will depend on the production, sequence plan sent by the central office. This problem deals with medium-sized instances and it is solved using online solvers. The contribution of this paper is a MILP model for the replenishment and inventory of the components in a car assembly line.

Key words: Integer Programming, Routing, Inventory, Assembly line.

1. Introduction

Today's customer looks for a specific configuration of cars; this has encouraged car manufacturers to offer plenty of options for each item of the cars.

Car manufacturers have changed from offering a single model to offering a huge number of model configurations. These car manufacturers have evolved from selling one model of one car as Ford did, with his Model-T, to offering many options (Ghosh and Gagnon, 1989).

For instance single visit to a car manufacturer's web page, such as Mercedes Benz, allows us to customized car, choosing each component such as rims, engine, tires, the design of the interior and exterior, steering, radio, safety, color, the size of engine, seats, and so on. This creates more theoretical configurations than actual ones that could be produced in one year.

Today's factories use car assembly lines in which the setup times between models can be ignored, and then the mixed model line approach is used.

This flexibility is provided by the development in the interactions between humans, machines, equipment, robots, transportation system, etc. In this paper, we are focusing on the interaction of routing and the replenishment of the components.

This flexibility increases the complexity of the replenishment of the component.

Assembly lines are flow-oriented production systems, which are still typical for the production of high quantity standardized commodities and they are even gaining importance in the low volume production of a customized product (Becker and Scholl, 2006). One of the most complex products that is built in the assembly lines is cars and trucks. The assembly lines are a way to mass-produce cars quickly and efficiently. They rely on the ability to assign easy tasks to humans and robots and move parts from one worker to another until the car is finished. Different tasks require certain equipment of machines, skills of workers, and components to be utilized. For the single model line (see fig.1), this was easy to solve because the requirements were periodic and homogeneous.

The place used to store the components next to the assembly line is limited. An increased use of the space necessitates reconfiguring the assembly line, or keeping the components in a different place implies that someone or something should do that additional task.

Providing the components as soon as they are needed creates many transportation problems and high cost for the factory.

The Oxford Dictionary defines “replenishment” as “restore (a stock or supply) to a former level or condition.” The core issue is determining what is the proper level and in which order the station will be replenished. This creates two problems; the inventory problem, and the routing problem.

The present work is a continuation of an early study on car assembly lines to explore the advantage of the joint decision in the assembly line.

In an earlier study (Pulido *et al.*, 2013) the importance of this decision was experimented for small instances. In this work, we make a comparison of two algorithms, in order to use big public instance we solve this problem using an online solver. The contribution of this paper is a joint model to decide the inventory and the routing of the assembly line.

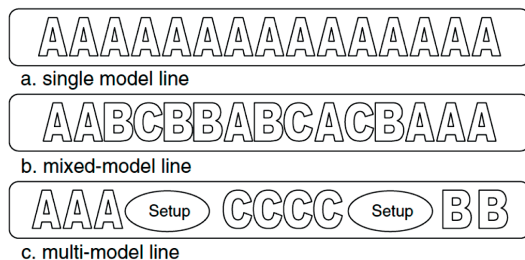


Figure 1. Different types of assembly lines source (Kazemi, 2011)

1.1. The Routing Problem

The delivery of the components to the workstations involves several transportation vehicles whose use and purchase affect directly on the cost. Then it is necessary to try to minimize the number of vehicles used and the distance that they travel. The vehicle routing problem was described by Laporte (1992) as the problem of designing optimal delivery or collection routes from one or several depots to a number of customers.

Campbell (1998) presents an extension called Inventory Routing Problem (IRP), which is based on the usage of products instead of orders. IRP deals with the repeated distribution of products to a set of

customers, taking into consideration the capacity of the vehicles and a penalty for the stock out. There is a different version of this problem with added features and adjustment for different types of industry, such as oil and gas (Gronhaug, 2010), or the use of genetic algorithms for a distribution network (Moin, 2011 and Archetti, 2012).

The IRP is the starting point for studying the integration of different components of the logistics value chain, i.e. inventory management and transportation (Campbell, 1998).

Nevertheless, those approaches do not take into consideration the deterministic consumption though the time (since the production sequence is known), nor the cost of the storage close to the assembly line.

1.2. The Inventory problem

The number of components that should be replenished for each customer (workstation) is one of the most studied questions in Operations.

The traditional inventory policies, such as Reorder Point, Min/Max, Lot for Lot or demand flow, or item location, are not suitable for this kind of problem since many types of the same components are installed in the same workstation, so the storage space is limited. Carrying zero inventory and stocking less production (Hall, 1983) is not possible because the replenishment time is constrained by the routes. In this problem it is assumed that the number of vehicles is lower than the number of workstations.

The replenishment of the car production line also presents some singularities since the size of some of the components is large, and many items depend on the type of car that is being assembled.

An excess of inventory induces an increase in the cost of interest on working capital, space cost, and risk of material obsolescence. A high inventory level in the assembly line is a big cost contributor. Some of the car manufacturer’s objectives are keeping low stock levels, performing the replenishment of the production line, and providing the required components at the right time (Monden, 1983). On the other hand, if there is a lack of components, there is a risk of incurring rework costs or even the stoppage of the line.

1.3. Integration of production and logistics

There are several papers in the literature that deal with the integration between production and logistics decisions at the strategic level, but almost

nothing has been done to integrate production and logistics problems at the operational level for daily decisions (Jin, 2008). Kaminsky (2003) proposed a two-stage model of the manufacturing supply chain, called the “2 Stage Production Distribution Problem” (2SPDP). Eskigun (2005) considers the outbound supply chain as the solution to minimize the fixed cost of facility location and transportation cost using a Lagrangian heuristic. The two key flows in such relationships are material and information. Prajogo (2012) addresses the integration of the relationship between material and information. Volling (2013) focuses on the planning of capacities and orders.

The contribution of this paper is a mixed integer model for the inventory routing problem of the mixed car model assembly line.

The remainder of the paper is organized as follows. Section 2 describes the problem. Section 3 provides a mathematical formulation of the problem with a detailed explanation of the assumptions. The computational experiments are presented in Section 4. In Section 5 the conclusions and further direction for research are given. Finally, the references are provided in Section 6.

2. Modeling assumptions

In the car assembly line being investigated, the production sequence has been decided for a planning period. The car has to go through N stations to be assembled. Each station installs a different type of components that need to be close to the assembly line before they are needed. All the components required for the production day are in one single warehouse. The transportation vehicles carry these components from the warehouse to the Stations (see Figure 2).

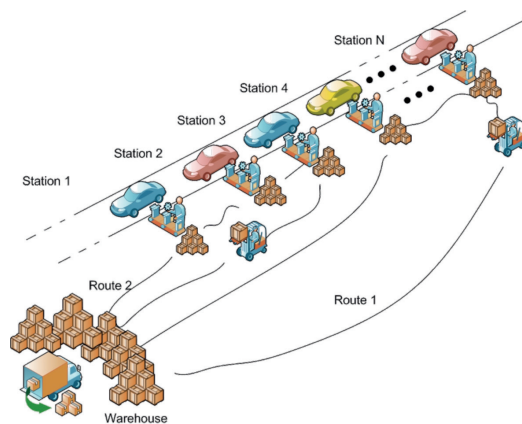


Figure 2. Description of the problem.

A “route” is defined as the course taken by a homogenous transportation vehicle and their arrival time to the workstations in order to get from the warehouse to the stations and back again. A transportation vehicle could have empty route.

The assembly line already exists and no changes to the production capacity or number of stations can be made.

Each model has a set of characteristics, such as engine, rims, tyres, steering, etc. These components could have different trims (Low or High). All the models are different from others models in at least one type of component (see Table 1).

Table 1. Type of different models

Model	Rims	Engine	...	Component n
A	Low	Low	...	Low
B	Low	Low	...	High
...
N	High	Low	...	High

The components required to assemble the products are storage next to the workstation. This storage space is capacitated. A holding cost will be imputed for every component that is storage in this area. As Grave (1987) suggest for a nondeterministic displacement times and the high cost of the line stoppage a safety stock is kept to deal with a possible delay in the replenishment of the components.

The transportation vehicles that bring the components to the stations are capacitated and homogeneous. An early arrival of the component causes space problems with the buffers of the production lines; a late arrival causes several problems in the production line. The components needed for the operation of a station are delivered as a kit. Dispatching only with a just-in-time policy increases the transportation cost and the green impact of the production line. It is necessary to select the route and the number of required components to get the lowest cost.

The final solution consists in designing routes for the transportation vehicles and the number of each components that need to transport in order to replenish the components minimizing the total cost.

The model contemplates safety stock to mitigate the risk due to any uncertainty; the level of the safety stock is determined by company policy and can be set to zero.

3. Problem Formulation

In this section, we begin by introducing the sets, parameters and decision variables (see Table 2). Later, we present the objective variable and the requirements.

The MIP problem minimizes the total cost of replenishment and inventory.

$$\begin{aligned} \min. & \sum_{r,l,l'} TC \times TDIS_{ll'} \times x_{rl'} + MC \sum_{j,l,l',r} f_{jll'r} + A \times \sum_r \alpha_r \\ & + \sum_j HC_j \times st_{j\tau l} \end{aligned} \quad (1)$$

The model is subject to the constraints' equation (2) to (26). Equation (1) is the objective function. Equations (2, 3, 4) ensure that each location is served by one route. Equation (5) ensures that the route has a predecessor except for the warehouse. Equation (6) force that if a route reaches a location, the route departs from that location. Equations (7, 8) set the number of routes equal to the number of vehicles. Equation (9) accounts a route if the vehicle visits at least one location. Equation (10) assigns first the vehicle number 1. Equation (11) limits the number of vehicles used to the available ones. Equation (12, 13) defines the time that arrival time for each location. Equation (14) constraint the amount of materials should be lower than the capacity. Equation (15) set the demand of certain characteristic only when the car required this characteristic.

Equation (16) defines that the amount of components that is left at the station. Equation (17) set the accumulated demand. Equations (18, 19) set that the accumulated components required. Equation (20) defines the stock. Equation (21) establishes the safety stock. Equations (22, 23, 24) establish that the required amount of components will be equal only to the replenished components when the replenishment occurs. Finally, equations (25, 26) define the time of the replenishment.

$$\sum_{r,l|l \neq l'} x_{rl'} = 1 \quad \forall l' \in L \setminus \{WH\} \quad (2)$$

$$\sum_{r,l|l' \neq l} x_{rl'} = 1 \quad \forall l \in L \setminus \{WH\} \quad (3)$$

$$\sum_r w_{rl} = 1 \quad \forall l \in L \setminus \{WH\} \quad (4)$$

$$w_{rl} = \sum_{l'|l' \neq l} x_{rl'}, \quad \forall r \in R, \forall l \in L \setminus \{WH\} \quad (5)$$

Table 2. Sets, parameters and decision variables

Index Set	
R	all homogenous transportation vehicles that could perform a route
L	all locations (workstations and warehouse)
M	all car model configurations
C	all car components
A	trim levels
J	characteristic $J \subseteq$ all car components \times trim levels
τ	discretized production time
Parameters	
R_{mj}	1 if the model $m \in M$ requires characteristic $j \in J$
$Y_{m\tau l}$	1 if model $m \in M$ is processed during cycle $\tau \in T$ in location $l \in L$
ST_{jl}	safety stock corresponding to characteristic $j \in J$ in location $l \in L$
ST_{0jl}	initial stock corresponding to characteristic $j \in J$ in location $l \in L$
A	amortization per transportation vehicle; it has to be paid if the transportation vehicle is used at least once
TC	traveling cost per distance unit
HC_j	unitary holding cost of component corresponding to characteristic $j \in J$ per time unit
MC	unitary moving cost of component
T	number of cycles to be planned $T = \lceil \tau \rceil$
CAP	maximum capacity of kits in a transportation vehicle
$TDIS_{ll'}$	displacement time from $l \in L$ to $l' \in L$
M	a large scalar value
Variables	
w_{rl}	1 if route $r \in R$ attends $l \in L$; 0 otherwise
$x_{rl'}$	1 if $l \in L$ immediately precedes $l' \in L$, on route $r \in R$; 0 otherwise
t_{rl}	discrete time in which the route $r \in R$ arrives to the location $l \in L$
$dem_{j\tau l}$	demand for component corresponding to characteristic $j \in J$ in cycle $\tau \in \tau$, in location $l \in L$
$dem_{j\tau l}^{ac}$	accumulated demand for the component corresponding to characteristic $j \in J$ at the beginning of cycle $\tau \in \tau$ in location $l \in L$
$c_{jl\tau r}$	amount of component replenished with characteristic $j \in J$ required in location $l \in L$, in route $r \in R$ in cycle $\tau \in \tau$.
$c_{js\tau r}^{ac}$	Accumulated amount of component replenished with characteristic $j \in J$ required in location $l \in L$, in route $r \in R$ in cycle $\tau \in \tau$.
$st_{j\tau l}$	Stock of component corresponding to characteristic $j \in J$ in location $l \in L$ at the beginning of cycle $\tau \in T$
α_r	1 if the route $r \in R$ is used for the replenishment; 0 otherwise
$\beta_{\tau l}$	1 if $t_{rl} = ord(\tau)$, 0 otherwise
$q_{jl'r}$	amount of component required with characteristic $j \in J$ in station $l' \in L$ in route $r \in R$
$f_{jll'r}$	The flow of component corresponding to characteristic $j \in J$ between l and $l' \in L$ in route $r \in R$

$$\sum_l x_{rll'} = \sum_l x_{rll} \quad \forall l' \in L, \quad \forall r \in R \quad (6)$$

$$\sum_{rl} x_{rlwh} = \sum_r \alpha_r \quad (7)$$

$$\sum_{rl'} x_{rwhl'} = \sum_r \alpha_r \quad (8)$$

$$\sum_{ll'} x_{rll'} \leq M \times \alpha_r \quad \forall r \in R \quad (9)$$

$$\alpha_r \geq \alpha_{r+1} \quad \forall r \in R \quad (10)$$

$$\sum_r \alpha_r \leq |R| \quad (11)$$

$$\text{if } l = wh \quad t_{rl'} \geq TDIS_{ll'} - M(1 - x_{rll'}) - M(2 - w_{rl-w_{rl'}}) \quad \forall r \in R, \forall l, l' \in L \quad (12)$$

$$\text{else } t_{rl'} \geq t_{rl} + TDIS_{ll'} - M(1 - x_{rll'}) - M(2 - w_{rl-w_{rl'}}) \quad \forall r \in R, \forall l, l' \in L \quad (13)$$

$$\sum_{jl} f_{jwhl} = CAP \quad \forall r \in R \quad (14)$$

$$dem_{j\tau l} = \sum_m R_{mj} Y_{m\tau l} \quad \forall j \in J, \forall \tau \in T, \forall l \in L \quad (15)$$

$$f_{jllr} - f_{jll'r} \geq q_{jll'r} - M(1 - x_{rll'}) - M(1 - x_{rll''}) - M(3 - w_{rl} - w_{rl'} - w_{rll''}) \quad \forall j \in J, \forall l, l', l'' \in L, \forall r \in R \quad (16)$$

$$dem_{j\tau l}^{ac} = dem_{j\tau-1l}^{ac} - dem_{j\tau l} \quad \forall j \in J, \quad \forall l \in L, \quad \forall \tau \in T \setminus \{1\} \quad (17)$$

$$\text{if } \tau = 1 \quad c_{jllr\tau}^{ac} = c_{jllr\tau} \quad \forall j \in J, \forall l' \in L, \forall r \in R, \quad \forall \tau \in T = \{1\} \quad (18)$$

$$\text{else } c_{jllr\tau}^{ac} = c_{jllr\tau-1}^{ac} + c_{jllr\tau} \quad \forall j \in J, \forall l' \in L, \forall r \in R, \quad \forall \tau \in T \setminus \{1\} \quad (19)$$

$$st_{j\tau l} = ST_{0j} - dem_{j\tau l}^{ac} + \sum_r c_{jllr\tau}^{ac} \quad \forall j \in J, \quad \forall \tau \in T \setminus \{1\}, \forall l \in L \quad (20)$$

$$st_{j\tau l} \geq ST_{jl} \quad \forall j \in J, \forall \tau \in T, \forall l \in L \quad (21)$$

$$c_{jllr\tau} \geq q_{jll'r} - M(1 - \beta_{rrl}) - M(1 - \sum_l x_{rll'}) \quad \forall j \in J, \quad \forall l' \in L, \forall r \in R, \forall \tau \in T \quad (22)$$

$$c_{jllr\tau} \leq q_{jll'r} \quad \forall j \in J, \forall l' \in L, \forall r \in R, \forall \tau \in T \quad (23)$$

$$c_{jllr\tau} \leq M \times \beta_{rrl} \quad \forall j \in J, \forall l' \in L, \forall r \in R, \forall \tau \in T \quad (24)$$

$$t_{rl} \leq \tau + M(1 - \beta_{rrl}) + M(1 - \sum_l x_{rll'}) \quad \forall r \in R, \quad \forall l' \in L, \forall \tau \in T \quad (25)$$

$$t_{rl} \geq \tau - M(1 - \beta_{rrl}) - M(1 - \sum_l x_{rll'}) \quad \forall r \in R, \quad \forall l' \in L, \forall \tau \in T \quad (26)$$

4. Computational Study

The modelling software AIMMS 3.13 was used and the standard solver Gurobi 5.5 was used to obtain the solution to the problem.

In order to deal with bigger instance the Gurobi was used at the NEOS server (Czyzyk, 1998, Gropp, 1997, and Dolan, 2001).

The specification of the neos-2 and neos-4 are Dell PowerEdge R410 servers with the following configuration:

- CPU - 2x Intel Xeon X5660 @ 2.8GHz (12 cores total), HT Enabled, 64 GB RAM.

For neos-3 and neos-5 are Dell PowerEdge R420 servers with the following configuration:

- CPU - 2x Intel Xeon E5-2430 @ 2.2GHz (12 cores total), HT Enabled, 64 GB RAM.

There is no public instances in the literature. The data for the experimentation was based on Car Sequencing instances from Regin & Puget (1997) instance #1, #2, and #3. The instance are public at Car Sequencing Problem Lib (www.csplib.org). From this sequence we make up the missing data. First we test the current instances, then we duplicate the number of stations (extended instances) keeping the same production ratio. Each instance has 100 cars.

A stopping criterion of 3600 sec was set for all the instances.

Table 3. Instances to be tested.

Instances	NCar	Mod	Stations
Regin & Puget #1	100	22	5
Regin & Puget #2	100	22	5
Regin & Puget #3	100	25	5
Regin & Puget #1(ext)	100	22	10
Regin & Puget #2(ext)	100	22	10
Regin & Puget #3(ext)	100	25	10

The holding cost, as was stated before, represents the cost of the opportunity to have space used to keep inventory instead of production activities.

In Table 4 the displacement time between stations is displayed. The acceleration, traveling time, deceleration, and the unloading of the components compose the displacement time. The traveling time is only relevant when the distance is greater than 5 stations.

We will compare the algorithm with a traditional constraint vehicle routing problem (CVRP) with optimal routes, keeping in consideration the production and the capacity of the vehicle.

Once the route is obtained the intrinsic cost of the inventory is calculated.

A fix cost for the use the transportation vehicle of \$266, plus a holding and moving cost of components.

Table 4. Displacement time between stations

Station	WH	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
WH		2	2	2	2	2	3	3	3	3	3
S1	2		2	2	2	2	2	3	3	3	3
S2	2	2		2	2	2	2	2	3	3	3
S3	2	2	2		2	2	2	2	2	3	3
S4	2	2	2	2		2	2	2	2	2	3
S5	3	2	2	2	2		2	2	2	2	2
S6	3	3	2	2	2	2		2	2	2	2
S7	3	3	2	2	2	2	2		2	2	2
S8	3	3	3	2	2	2	2	2		2	2
S9	3	3	3	3	2	2	2	2	2		2
S10	3	3	3	3	3	2	2	2	2	2	

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A fix cost for the use the transportation vehicle of \$266, plus a holding and moving cost of components.

We are going to present the results from the six instances in section 4.3. For comparison details we are going to examine the instance Regin & Puget #1(ext). This instance has 100 cars, with 22 types of cars, and it will be produced in 10 stations.

The experimentation will run with different ratio of traveling cost and holding cost.

4.1. Routing Analysis

In Tables 5 and 6 we show the arrival time of the transportation vehicles to the stations. Table 5 uses the compound approach, and Table 6 uses a classical CVRP.

Table 5. Arrival time of the transportation vehicles (instance Regin & Puget #1).

Station	WH	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
V1	15			13	11			3	5	8	
V2	23	15	21			2	13				10
V32											

Table 6. Arrival time of the transportation vehicles of a classical CVRP (instance Regin & Puget #1).

Station	WH	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
V1	25	12	19	21	23	14					16
V2	28						15	25	19	23	
V3											

Both routes use only two vehicles; the joint approach uses the first vehicle to deliver the urgent components and dispatch the second vehicles later, reducing the cost.

4.2. Inventory Analysis

In Figures 3 and 4, the inventory levels of the station 3 are displayed. The holding cost of the instances displayed for high trim is 20 cents per minute and 10 cents per minute for the low trim.

The model adjusts the replenishment in order to minimize the area below the line.

The replenishment is done as soon as the station reaches the safety stock, e.g. the arrival at station 3 happens at minute 21 instead of minute 13. This delay of 8 time units represents 15% savings in the holding cost (see Table 7). The safety stock lays an important role in the cost; it is space and money that we have dedicated to avoid logistic problems.

The maximum stock in the assembly line also decreases from 61 to 56 for low trim and from 25 to 22 units for high trim; this decrease of 8 units represents 10% of savings in space that can

be allocated to other production activities. The inventory level is always above the safety stock.

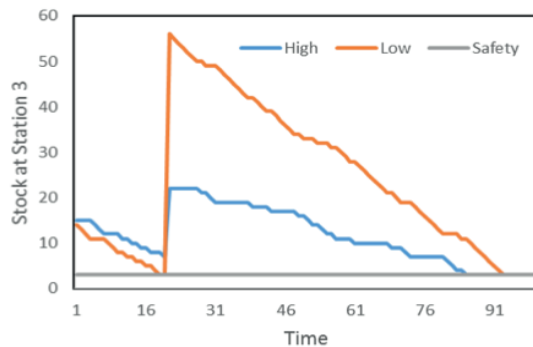


Figure 3. Stock and Safety Stock at Station 3 of the instance joint model (instance Regin & Puget #1(ext))

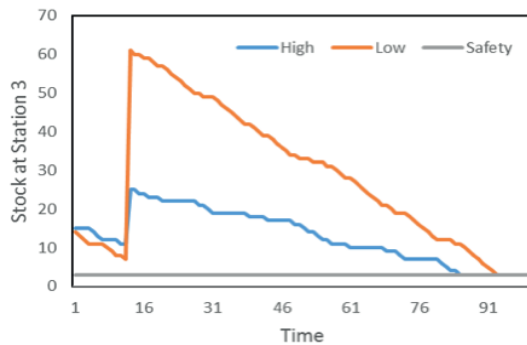


Figure 4. Stock and Safety Stock at Station 3 of the instance (instance Regin & Puget #1(ext)) of CVRP

Table 7. Cost of the stock in the Station 3 (instance Regin & Puget #1(ext)).

	Join Model	CVRP	Diff(%)
High trim	172.4	196.4	13.9
Safety Stock	60	60.0	0.0
sub total	232.4	256.48	10.3
Low trim	203.6	246.8	21.2
Safety Stock	30.0	30.0	0.0
sub total	233.6	276.8	18.5
Total	466	533.2	14.4

4.3. Cost Analysis

Table 9 shows a comparison between the costs of the 3 instances. In all the instances, we obtain savings due to the joint decision, taking into consideration the most suitable time to replenish; instead only the shortest path could provide interesting savings only by changing the route.

The model of this system has 4 costs (see eq.1); the fixed cost for the use of a transportation vehicle (A), the cost of the distance traveled (TC), the cost of carrying the load (MC) and the holding cost (HC). Changing the cost of any of the parameters will reflect the routing and replenishment routes of the company.

Table 8. Comparison of total cost (instance Regin & Puget #1(ext)).

	Join Model	CVRP	Diff(%)
Route	203.6	246.8	-3.24%
Inventory Cost	30.0	30.0	11.42%
Total	7111	7676	7.99%

Table 9. Comparison of the results of the two approaches.

Instance	MC/ HC	N Mod	N Car	N Loc	N Var cont	N Int Var	Obj Joint	CVRP	HC	Total CVRP+HC	Diff (%)
Regin & Puget #1	0.5	22	100	5	4500	1611	29364	828	32403	33231	13.2
Regin & Puget #2	0.5	22	100	5	4500	1611	28966	828	30843	31671	9.3
Regin & Puget #3	0.5	25	100	5	4554	1611	28721	828	31722	32550	13.3
Regin & Puget #1(ext)	0.5	22	100	10	13548	3366	56151	1611	60687	62298	10.9
Regin & Puget #2(ext)	0.5	22	100	10	13548	3366	55457	1611	56624	58235	5.0
Regin & Puget #3(ext)	0.5	25	100	10	13575	3366	57311	1611	65297	66908	16.7
Regin & Puget #1	5	22	100	5	4500	1611	3678	828	3337	4165	13.24
Regin & Puget #2	5	22	100	5	4500	1611	3657	828	3240	4068	11.24
Regin & Puget #3	5	25	100	5	4554	1611	3615	828	3084	3912	8.22
Regin & Puget #1(ext)	5	22	100	10	13548	3366	7111	1611	6068	7679	7.99
Regin & Puget #2(ext)	5	22	100	10	13548	3366	6874	1611	5662	7273	5.80
Regin & Puget #3(ext)	5	25	100	10	13575	3366	7047	1611	6134	7745	9.90
Regin & Puget #1	50	22	100	5	4500	1611	1117	828	334	1162	4.0
Regin & Puget #2	50	22	100	5	4500	1611	1111	828	324	1152	3.7
Regin & Puget #3	50	25	100	5	4554	1611	1110	828	309	1137	2.4
Regin & Puget #1(ext)	50	22	100	10	13548	3366	2149	1611	607	2218	3.2
Regin & Puget #2(ext)	50	22	100	10	13548	3366	2133	1611	567	2178	2.1
Regin & Puget #3(ext)	50	25	100	10	13575	3366	2161	1611	652	2263	4.7

A good example of the problem (see Table 8) of consider the problem separately is that CVRP select the best route for all the instances of the same number of stations for the same total demand of components. However, the demand over the time is different and consequently the stocks are different.

When the moving cost is more representative for the model ($MC > HC$), savings decrease since the CVRP achieves the optimal, and the impact on the holding cost is not so important. On the other hand, when the holding cost become more important ($MC < HC$) this model present bigger savings that the separate decision. For all the instance we obtain a better result with the joint decision that with the separate approach.

All the CVRP and HC problems were solved up to optimality, for the joint model only the small instance and the instance that get a ratio of HC/MC of 50 was solved to optimality. The extended instances required the stopping criterion of one hour.

In the table 10 the result of ANOVA analysis are displayed, we set the type of instance and the different results and we obtain a $p=0.950$ then we assume that the result are the same for the different instances, since the moving cost of the components is the same for high or low trim, and the only difference is the difference of the holding cost of the components.

Table 10. ANOVA comparison among different mix of demands.

Source	DF	SSC	MS	F	P
Instance	2	408247	204124	0.00	0.999
Error	15	3.23 E+9	2.16E+E8		
Total	17	3.23 E+9			

s=14688, r-sq=0.01%, r-sq(adj)=0.00%

5. Conclusions

In this work, the inventory and the routing problems have been solved jointly. The routing model should consider more factors than just the transportation cost; also, the inventory should consider more factors than replenish when a level is reached. The main factor in the delivery of material should not only be the decrease of the transportation costs but also the decrease of the holding cost of the components.

The selection of the routes and the inventory levels should consider the specific requirement of materials over the time to decrease the cost.

The cost of the space is an amplifier of the savings of the model. When there are restrictions in the space closest to the assembly line, the model tries to keep the lowest inventory along the planning period. The replenishment is made before the inventory level reaches the safety stock. Following the Lean idea, it is possible to decrease the safety stock until it reaches zero safety stock, always keeping in mind the risk of any delays with the consequence of the stoppage of the assembly line.

5.1. Future Research

As this is a NP hard problem, many research directions come up. The first one is to try to change the sequence in order to create a joint algorithm to find a joint solution; additionally, a metaheuristic needs to be developed to solve bigger problems.

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